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## A Charm of Puzzles. The Fate of Richard Kilvington's Philosophical Ideas

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## A CHARM OF PUZZLES. THE FATE OF RICHARD KILVINGTON'S PHILOSOPHICAL IDEAS

The so-called *science of motion* is widely recognized as the most important achievement of fourteenth-century natural philosophers<sup>1</sup>. Many historians of medieval science still claim that Thomas Bradwardine was the first philosopher who reformulated in a strict mathematical manner *laws of motion* presented in the last part of the book VII of Aristotle's *Physics*<sup>2</sup>, exploiting for this purpose Eudoxean calculus of ratios<sup>3</sup>. After Bradwardine, many Oxford thinkers adopted the above-mentioned mathematical tool to analyse different, more or less complicated cases – regarding not only local motion. Medievalists distinguish them as the school of *Oxford Calculators*<sup>4</sup>. Anneliese Maier and Alistair Crombie tried to prove that ideas developed by those thinkers influenced Galileo's inventions and in this sense contributed to the Scientific Revolution<sup>5</sup>. Nowadays, historians of science discard this thesis, pointing out the fact that medieval philosophers' aim was always to correct or rather – as some of those philosophers said<sup>6</sup> – to understand properly Aristotle's statements and not to give a mathematical description of observable phenomena<sup>7</sup>. On the other hand, the introduction of mathematics into natural philosophy can be recognized as a step towards modern science<sup>8</sup>.

<sup>1</sup> See E. Grant, *The Foundations of Modern Science in the Middle Ages. Their Religious, Institutional, and Intellectual Contexts*, Cambridge 1996.

<sup>2</sup> See Aristotle, *Physics* 249b29–250b10.

<sup>3</sup> See J. A. Weisheipl, OP, *The Interpretation of Aristotle's 'Physics' and the Science of Motion in: The Cambridge History of Later Medieval Philosophy*, N. Kretzmann, A. Kenny, J. Pinborg (eds), Cambridge 1982, pp. 533–536.

<sup>4</sup> See E. D. Sylla, *The Oxford Calculators in: The Cambridge History of Later Medieval Philosophy*, pp. 540–541.

<sup>5</sup> See A. C. Crombie, *Medieval and Early Modern Science*, vol. 2: *Science in the Later Middle Ages and Early Modern Times: XIII–XVII Centuries*, Cambridge (Mass.) 1963, A. Maier, *Die Vorläufer Galileis im 14. Jahrhundert. Studien zur Naturphilosophie der Spätscholastik*, vol. 1, Rome 1949.

<sup>6</sup> See e.g. Guilielmus Ockham, *Expositio super libros Physicorum, Libri IV–VIII*, R. Wood, R. Green, G. Gál et al. (eds), New York 1985 (*Opera philosophica*, t. V), VI, 13, 3, p. 562, v. 21–22: *Est primo declarandum quod Philosophus intendit ponere infinitas partes continui actualiter existentes in rerum natura*.

<sup>7</sup> See J. E. Murdoch, E. D. Sylla, *The Science of Motion in: Science in the Middle Ages*, D. Lindberg (ed.), Chicago 1978, pp. 246–251.

<sup>8</sup> See E. Grant, *The Foundations of Modern Science in the Middle Ages*.

But let me leave this controversy aside. What I want to focus on here is the question why fourteenth-century natural philosophers paid their attention to some ideas and not to other ones. Surely, their criterion was not the accuracy between description and reality. For example, it is easy to notice that the famous *Bradwardine's law*, presented in the *Tractatus de proportionibus velocitatum in motibus* (1328), does not determine the actual speed of a body, even if forces are given, nor does it describe properly change of speed when forces change<sup>1</sup>. Nevertheless, William Heytesbury, Richard Swineshead, Jean Buridan, Nicolas Oresme and many other medieval philosophers employed this *law* in their treatises<sup>2</sup>.

In my opinion, another medieval thinker's ideas – namely Richard Kilvington's, can be instructive here. Kilvington was Thomas Bradwardine's *socius* at Oxford<sup>3</sup>. What is more, thanks to Elżbieta Jung's recent studies we know that Kilvington was the first to reformulate Aristotle's *laws* of motion using Eudoxean calculus of ratios. Bradwardine in his *Tractatus de proportionibus* just gave Kilvington's statements a more refined and elegant form<sup>4</sup>. Actually, Kilvington did not write any separate treatise intended as a systematic presentation of his theories. Instead, he strewed his novel ideas in arguments scattered all over his commentaries on Aristotle's works<sup>5</sup>. Although this feature of Kilvington's method of scientific inquiry impedes the reconstruction of specific theories that underlie his statements, it serves my present purpose. Because of their complexity, Kilvington's commentaries could not be used as handbooks or merely as a source of complete solutions. Therefore, his contemporaries and followers who presented and discussed his arguments must have recognized them as important or intriguing.

### Infinite sets and subsets

The ingenuity of the first of Richard Kilvington's arguments I present in my paper strikes even a modern reader. In one of his questions on Peter Lombard's *Sentences* (written ca. 1332–1334) Kilvington demonstrated the properties of infinite multitudes in a way that resembles Georg Cantor's exposition developed more than a half millennium later<sup>6</sup>. In Kilvington's question *Utrum unum infinitum potest esse maius alio (Whether one infinity can be greater than another?)* we find, for example, the following argument: Let us give one crown to everyone out of infinitely many men. Now, we are

<sup>1</sup> See E. Jung *olim* Jung-Palczewska, *Między filozofią przyrody a nowożytnym przyrodoznawstwem. Ryszard Kilvington i fizyka matematyczna w średniowieczu*, Łódź 2002, pp. 285–287.

<sup>2</sup> See E. D. Sylla, *The Oxford Calculators*, pp. 555–562, E. D. Sylla, *Transmission of the New Physics of the Fourteenth Century from England to the Continent* in: *La nouvelle physique du XIV<sup>e</sup> siècle*, S. Carotti, P. Souffrin (eds), Firenze 1997, pp. 92–109.

<sup>3</sup> See E. Jung, *Works by Richard Kilvington* in: *Archive d'histoire doctrinale et littéraire du Moyen Age* 67, 2000, pp. 181–183.

<sup>4</sup> See E. Jung, *Między filozofią przyrody ...*, pp. 96–101.

<sup>5</sup> See E. Jung, *Między filozofią przyrody ...*, pp. 37–38.

<sup>6</sup> G. Cantor, *Ein Beitrag zur Mannigfaltigkeitslehre* in: *Gesammelte Abhandlungen mathematischen und philosophischen Inhalts*, E. Zermelo (ed.), Berlin – Heidelberg – New York 1980, p. 119.

able to show that even after destroying infinitely many crowns there will remain the same, infinite number of them, because: *when the first crown is destroyed, the first man gets a crown of the second man, and the second gets the crown of the third, and so on*<sup>1</sup>.

It is clear that Kilvington was convinced that an infinite set of men is in a sense equal to an infinite set of crowns, and as such it would be equal even when we took away one or more crowns (cf. fig. 1).

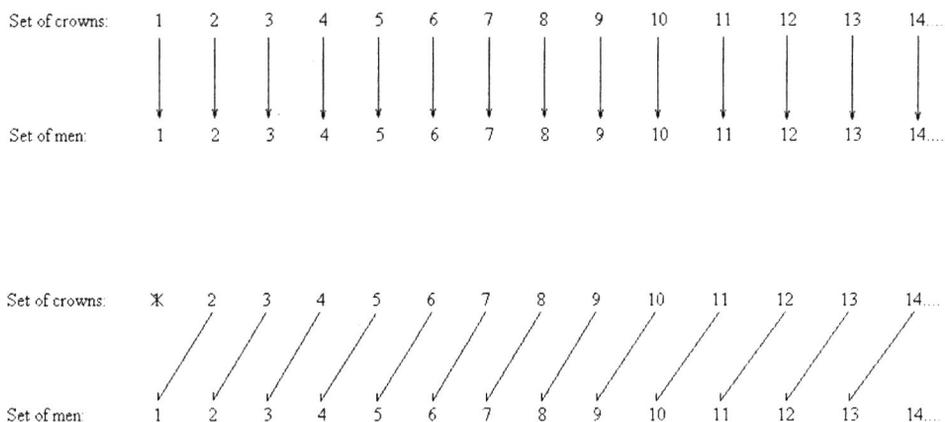


Fig. 1

Obviously, the same concept underlies another argument of Kilvington's:

*Let A be a relative infinite limited in the place B and let C be an infinite juxtaposed with A or placed equally in such a way that C does not exceed or is exceeded by [A]. Now, I take that in the first proportional part<sup>[2]</sup> of some time God destroys the first one-foot quantity of A, and in the second proportional part of this period the second one-foot [quantity] and so on infinitely. And with respect to body C, when God destroys the first one-foot quantity*

<sup>1</sup> Ricardus Kilvington, *Utrum unum infinitum potest esse maius alio*, Ms. Vat. lat 4353, f. 40v: *Dando primi homini primam coronam et secundi homini secundam coronam et sic deinceps secundum omnes partes proportionales unius horae. Et ita econtra posset adnihilari numerus infinitus coronarum per consimiles adnihilationes coronarum secundum partes proportionales in aliquo tempore, sic videlicet quod in prima parte corrumpatur vel adnihiletur unius corona et in secunda secundi corona et sic deinceps. Sed probatur quod non, quia adnihilata prima corona ponatur quod primus homo capiat coronam secundi homini et secundus capiat coronam tertii et sic deinceps, et sequitur quod per totam horam et in fine horae erunt tot coronae, quot in principio.* (All translations of Latin passages in this article are mine – R. P.)

<sup>2</sup> By proportional parts of some quantity or period medieval thinkers understood the series of parts that are in the same proportion to the remaining whole – usually they meant successive halves, i.e. a half, a quarter, an eighth, etc. Kilvington used the number of proportional parts of a continuum as an example and a gauge of actual infinite multitude.

*of A, at the same time [he] destroys the second one of C, and when he destroys the second [one-foot quantity] of A he also destroys the fourth one of C, and similarly when he destroys the third [part] of A he also [destroys] the sixth of C, and so on infinitely, so there remain the alternate one-foot long parts of C. Thus, in the end of this time the whole of A will be destroyed [...] and C will not be wholly destroyed, but [...] C will be the same as it was before.<sup>1</sup>*

In short, Kilvington argues that there are as many even, one-foot long parts in an infinitely long line as all the parts of the same longitude in the same line. One cannot help thinking that this is just a complicated way of saying that the infinite set of natural numbers is equal to its proper subset of even numbers – that is also infinite (cf. fig. 2).

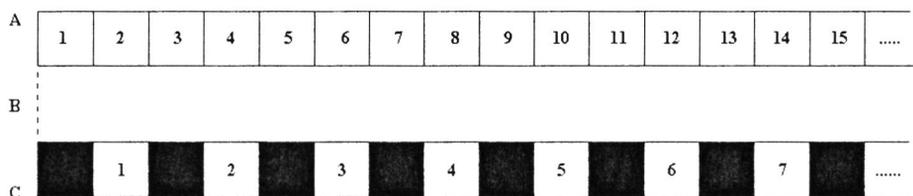


Fig. 2

In his question Richard Kilvington included more arguments of this kind. But I think, the above ones suffice to state that Kilvington must have been convinced that an infinite set can be put in one-to-one correspondence with its infinite subset. And it is worth noting that this paradoxical feature of infinite sets nowadays serves as a criterion of determining them.

In Thomas Bradwardine's monumental treatise *De causa Dei* written ca. 1344 in the context of the discussion on the possible eternity of the created world one finds the same – just slightly reformulated – arguments (cf. fig. 3). For example, we read there:

*Let A be the whole (i.e., infinite) multitude of souls, and B the whole (infinite) multitude of bodies; therefore each single unity of multitude A evenly corresponds to a single unity of B, and whole [multitude A] to whole [multitude B], and vice versa [...]. And that can be clearly demonstrated when*

<sup>1</sup> Ricardus Kilvington, *Utrum unum infinitum potest esse maius alio*, f. 39v–40r: *Sit A secundum quid infinitum terminatum ad B situm et sit C unum infinitum iuxta positum A vel suppositum aequaliter sic quod C nec excedat nec excedatur. Et pono quod Deus corrumpat in prima parte proportionali alicuius temporis primam pedalem quantitatem de A et in secunda parte proportionali eiusdem <temporis> secundam pedalem et sic in infinitum. <Et etiam pono> quod de C corpore corrumpat Deus quando corrumpit primam pedalem de A secundam de C, et quando corrumpit secundam de A corrumpat quartam de C, et sic quando tertiam de A sextam de C et sic in infinitum, ita quod alternae partes de C pedalis quantitatis maneant. Tunc in fine temporis totum A corrumpetur [...] et C non totaliter corrumpetur, sed [...] C est tantum, quantum prius fuit.*

*distributing the souls either by the omnipotence of God, or by imagination, that way: the first soul to the first body, the second one to the second, and so on. In effect of this distribution any soul will have its unique body, and any body its unique soul.*<sup>1</sup>

And a little further:

*Assuming [multitudes] A and B like before, let us give the second soul to the first body, and the fourth to the second, and the sixth to the third, and so on as long as there are available souls, always alternating in the multitude A, yet in the multitude B proceeding continuously. Eventually, each and every body in the multitude B will be animated and still an infinity of souls will remain.*<sup>2</sup>

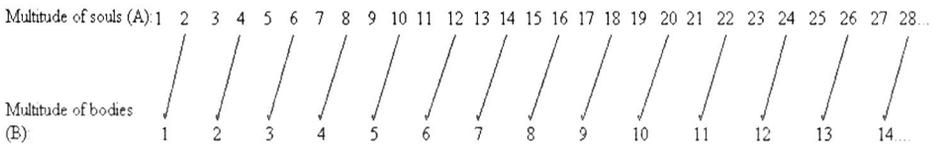
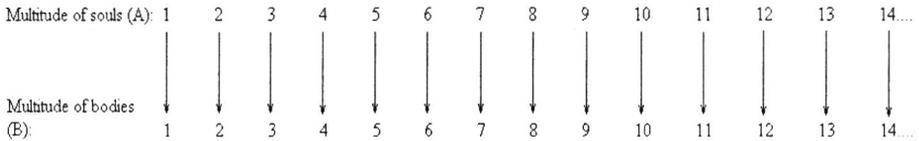


Fig. 3

It should be mentioned here that Bradwardine used this reasoning primarily against the Aristotelian concept of eternal and uncreated world, which was then believed to presuppose the existence of infinitely many souls and bodies<sup>3</sup>. Eventually, using the same method of pairing elements of one infinite set with elements of other one, Bradwardine arrived at the conclusion that if the world were eternal God might have created only popes or saints, and there would be

<sup>1</sup> <Thomas Bradwardine>, *Thomae Bradwardini Archiepiscopi olim Cantuariensis De causa Dei contra Pelagium et de virtute causarum ad suos Mertonienses*, Londini 1628, 121E.

<sup>2</sup> <Thomas Bradwardine>, *Thomae Bradwardini Archiepiscopi olim ...*, 122B.

<sup>3</sup> See J. M. M. H. Thijssen, *The Response to Thomas Aquinas in the Early Fourteenth Century* in: *The Eternity of the World – in the Thought of Thomas Aquinas and His Contemporaries*, J. B. M. Wissink (ed.), Leiden – New York – København – Köln 1990, p. 85.

still an infinite number of them<sup>1</sup>. Why, then, has the Omnipotent not done that – one could ask rhetorically ...

In the same treatise we find another argument that sounds familiar. Let us suppose, Bradwardine argues, that God pays everyone out of infinitely many men one penny for one-day work. Still, on the one hand, he claims, it is possible that one of them is paid one thousand pence for the same work. On the other hand, it is possible that there remain infinitely many pence in God's treasury. *Who, then, would be so greedy as to take all that money?*, asked Bradwardine in the end<sup>2</sup>.

It is worth noting that despite the fluency in using the method of pairing elements of infinite sets, Bradwardine clearly considered all derived conclusions as too paradoxical to be true<sup>3</sup>. In the passage from *De causa Dei* I am referring to now, Thomas Bradwardine strove to discredit not only the Aristotelian theory of the eternity of the world, but also the concept of infinity of a certain *falsigraphus* – a deceptive writer<sup>4</sup>. That one, most likely, was Richard Kilvington<sup>5</sup>. Therefore, it was probably Bradwardine's merit that Kilvington's conception of the relation between infinite sets and subsets was consigned to oblivion<sup>6</sup>. Still, another reasoning of Kilvington's related to infinity enjoyed popularity among medieval philosophers.

### Linea girativa

In one of Richard Kilvington's questions on Aristotle's *De generatione, Utrum continuum sit divisibile in infinitum* (written ca. 1324–1325), we find the following argument: Take a column and mark out all its proportional parts – a sequence of halves of its height. Then we draw a spiral line circumscribing this column, starting from a point on a circumference of its base, so that each succeeding coil embraces one proportional part, i.e. the first coil the first half of the column, the second coil one fourth, the third coil one eighth of the column, and so on *in infinitum*. It is obvious that each coil is longer than the circumference of the column, and there are infinitely many coils forming a continuous line. Consequently, the line is actually infinitely long, for it can be regarded as a sum of infinitely many parts, each of them possessing a certain longitude<sup>7</sup> (cf. fig. 4.).

<sup>1</sup> <Thomas Bradwardine>, *Thomae Bradwardini Archiepiscopi olim ...*, 123A.

<sup>2</sup> <Thomas Bradwardine>, *Thomae Bradwardini Archiepiscopi olim ...*, 123C–124B.

<sup>3</sup> <Thomas Bradwardine>, *Thomae Bradwardini Archiepiscopi olim ...*, 123C.

<sup>4</sup> <Thomas Bradwardine>, *Thomae Bradwardini Archiepiscopi olim ...*, 123C & 131D.

<sup>5</sup> See R. Podkoński, *Thomas Bradwardine's Critique of 'Falsigraphus's' Concept of Actual Infinity* in: *Studia Antyczne i Mediewistyczne* 36, 1/2003, pp. 141–153.

<sup>6</sup> For example, Galileo was astounded when he discovered in 1638 that there are as many natural numbers as their squares. See Galileo Galilei, *Discorsi e dimostrazioni matematiche intorno à due nuove scienze*, Leiden 1638, pp. 78–79.

<sup>7</sup> Ricardus Kilvington, *Utrum continuum sit divisibile in infinitum*, edited by R. Podkoński in: *Mediaevalia Philosophica Polonorum* 36, 2/2007, pp. 136–137.

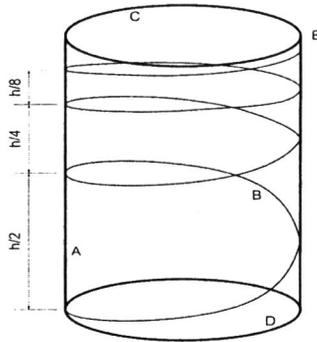


Fig. 4

In his reasoning, Kilvington raised doubts about the *upper* limit of the spiral line that, circumvolving the column, should have both extremes and as such it seems finite. He argued that this line has to be immediately adjacent to the upper surface of the column, because if it was not, there would be some proportional parts of the column not circumvolved by the line. Consequently, the spiral line would be finite, because it would consist only of a finite number of coils. But if a spiral line is immediately adjacent to the upper surface, it should be possible to label its end point. And if there was an end point the line would be finite, which is against the main proposition of this argument. Finally, Kilvington proved that although there is *no distance* between the spiral line and the upper surface of the column, there is also no determined point ending the line. In fact, he observed, one can consider any of the points on the circumference of the upper surface of the column as an end point of the spiral line. If so, the spiral line has an infinite number of end points. Therefore, there is no determinable end point, and the line is infinite and immediate to the circumference of the upper surface of the column<sup>1</sup>.

Eventually, Kilvington affirmed the last conclusion saying that the spiral line has two limits. One of them is intrinsic – and this is the starting point of the line. The other, however, is an extrinsic limit – and this is the circumference of the upper surface of the column, which does not belong to this line<sup>2</sup>. The only possible explanation is that Kilvington considered the spiral line to approach this circular line asymptotically (cf. fig. 5):

<sup>1</sup> See Ricardus Kilvington, *Utrum continuum sit divisibile in infinitum*, pp. 138–140.

<sup>2</sup> See Ricardus Kilvington, *Utrum continuum sit divisibile in infinitum*, p. 166.

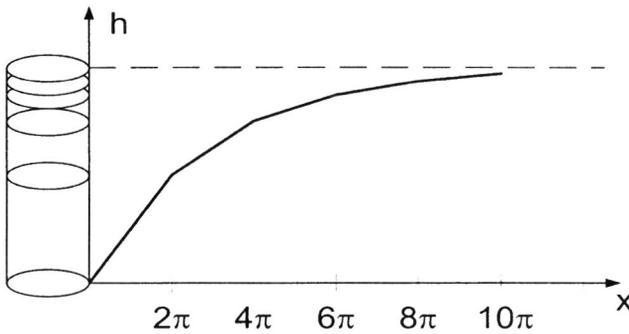


Fig. 5

Kilvington repeated the above construction in his *Sentences*' commentary in the above-mentioned question *Utrum unum infinitum potest esse maius alio*. The difference was that there he constructed two spiral lines, both starting from the same point in the middle of the height of the column and going into opposite directions toward its upper surface and base. The laconic manner of presenting this construction suggests that Kilvington presumed his audience to be familiar with this reasoning. As it was already proven in the question *Utrum continuum ...*, both halves of the line would lack an end point, and consequently the whole line would be infinite *utroque extremo*<sup>1</sup>.

The above argument is echoed in Thomas Bradwardine's *Sentences*' commentary dated ca. 1332–1334. Bradwardine, however, limited himself to flatly refusing Kilvington's conclusions giving no good reason<sup>2</sup>. The same example of infinite spiral line was later exploited by Roger Rosethus in his commentary on the *Sentences* (ca. 1335)<sup>3</sup>, by John Buridan in his *Physics* commentary (ca. 1350–1357)<sup>4</sup>, Albert of Saxony in his *Physics*<sup>5</sup>, Marsilius of

<sup>1</sup> See Ricardus Kilvington, *Utrum unum infinitum potest esse maius alio*, f. 40r–v: *Linea sit infinita utroque extremo, ut patet de linea girativa in corpore columnari quae per utriusque suae medietatae girat singulas partes proportionales versus extrema illius corporis. Et quod talis sit infinita patet, quia additio fuit sibi per partes aequales in infinitum, igitur ibi est infinitum tale et in actu.*

<sup>2</sup> See Thomas Bradwardine, *Utrum in entibus possit esse aliquod infinitum*, Ms. Paris, Bibl. Nat. lat. 15805, f. 40ra: *Probatur quod sic. Primo in quantitate continua, quia linea gyrativa aliqua vel illa quae incipit in basi gyrando per partes proportionales, est infinita. Sed non sequitur quia terminatur in basi. Contra: incipiat in medio et procedat versus utrumque extremorum proportionaliter. Dices quod adhuc terminatur extrinsece ad terminos columnae[...].*

<sup>3</sup> See Roger Roseth, *Lectura super Sententias*, Quaestio 5, articulus 2: *Utrum aliqua creatura possit esse infinita* in: Roger Roseth, *Lectura super Sententias. Quaestiones 3, 4 & 5*, edited by O. Hallamaa, Helsinki 2005, pp. 266–272.

<sup>4</sup> Johannes Buridanus, *Utrum linea aliqua gyrativa sit infinita, et semper accipio infinitum categorematicae*, in: *John Buridan's Tractatus de infinito*, edited by J. M. M. H. Thijssen, Nijmegen 1991, pp. 23–27.

<sup>5</sup> Alberti de Saxonia *Quaestiones super libros de Physica auscultatione Aristotelis*, Venetiis 1516, lib. III, quaest. XI, f. 39b.

Inghen in his *Physics* commentary (before 1396)<sup>1</sup>, Benedictus Hesse in *Physics* (ca. 1415)<sup>2</sup>, and in the 16<sup>th</sup> century by Johannes Mair in his *Tractatus de infinito*<sup>3</sup>.

Perhaps the most comprehensive analysis of the properties of a spiral line is provided by John Buridan. Buridan formulated for this purpose a separate question: *Utrum linea aliqua girativa sit infinita, et semper accipio infinitum cathegorematicce* (*Whether a certain spiral line is infinite – and I always understand [the term] ‘infinite’ cathegorematically*)<sup>4</sup> never mentioning Kilvington's name. The question begins with a detailed description of a construction of an infinite spiral line. At the end of this part Buridan concluded that taking into account the Aristotelian concept of continuity<sup>5</sup> one must admit that this spiral line is actually infinitely long<sup>6</sup>. It contradicts, however, Aristotle's statement, that actual infinities cannot exist<sup>7</sup>. *To me this question is difficult* – Buridan stated there openly<sup>8</sup>.

In the following parts of his question Buridan clearly strove to negate the conclusion that a spiral line is infinitely long. One must notice, however, that his analyses, unlike Bradwardine's and Rosethus's, are logically and methodologically coherent. Several times Buridan remarked that it is really difficult to determine properly the properties of a spiral line<sup>9</sup>. But, eventually, he presented the following argument: let us imagine a body that touches the upper basis of a column. If we admit that a spiral line is tangent to this body, we must agree that the line has both limits – which means that it is finitely long. But in this case we also must admit that there are only finitely many proportional parts of a column, and that the upper basis is the last proportional part of this column. This conclusion, however, contradicts Aristotle's concept of continuity. Therefore, said Buridan, we must accept that a spiral line is not tangent to the above-mentioned body. If it is so, then there is some distance between this body and the end of the spiral line. Consequently, there are some proportional parts of the column not circumscribed by the line. In effect, Buridan concluded, the spiral line consists only of finitely many *coils*, which means it is only finitely long<sup>10</sup>.

<sup>1</sup> See <Marsilius de Inghen>, *Abbreviationes libri Physicorum edite a prestantissimo philosopho Marsilii Inghen*, Pavia ca. 1490, fol. sign. d, col. d; fol. sign. d2, cols. a, b.

<sup>2</sup> Cf. Benedictus Hesse, *Utrum aliqua linea girativa sit infinita, accipiendo “infinitum” cathegorematicce*, in: *Quaestiones super octo libros ‘Physicorum’ Aristotelis*, edited by S. Wielgus, Wrocław - Warszawa - Kraków et al. 1984, pp. 384–387.

<sup>3</sup> <Johannes Mair>, *Propositum de infinito Magistri Johannis Maioris in: Le traité «De l’infini» de Jean Mair*, edited and translated by H. Elie, Paris 1938, pp. 12–52. It is worth noting that Mair description of a spiral line is that it circumscribes a column: *modo serpentis in arbore Adae* (like a serpent on the Adam's tree).

<sup>4</sup> See n. 4 on the preceding page.

<sup>5</sup> Aristotle, *Physics* 232b.

<sup>6</sup> Johannes Buridanus, *Utrum linea aliqua gyrativa sit infinita ...*, pp. 23–25.

<sup>7</sup> Aristotle, *Physics* 206a–b.

<sup>8</sup> Johannes Buridanus, *Utrum linea aliqua gyrativa sit infinita ...*, p. 25.

<sup>9</sup> Johannes Buridanus, *Utrum linea aliqua gyrativa sit infinita ...*, pp. 26, p. 27 & p. 29.

<sup>10</sup> Johannes Buridanus, *Utrum linea aliqua gyrativa sit infinita ...*, pp. 30–31.

As presented above, the problem of determining the upper end of the line played a central role in Richard Kilvington's reasoning. Although Buridan seems to have rejected Kilvington's view, surprisingly, at the end of his question he admitted that if a spiral line circumscribed all proportional parts of a column it would actually be infinitely long. *But there is no such line* – he remarked eventually<sup>1</sup>. As I have stated above, Buridan was not the last medieval philosopher who discussed the properties of an infinite spiral line. Nevertheless, all later thinkers identified this construction with Buridan's name<sup>2</sup>.

## Conclusions

Twentieth-century Italian philosopher Giorgio Colli in his book entitled *La nascita della filosofia* underlines the importance of puzzles and paradoxes for the development of ancient Greek philosophy<sup>3</sup>. His general thesis is that the first philosophers conducted a kind of intellectual competition by either inventing puzzles or trying to solve ones. Of course, it is just one of the possible scenarios of the birth of Western philosophy, but, I think, a very attractive and well-grounded one. For example, Plato's dualism of ideal and phenomenal worlds can be seen as an answer to Parmenides of Elea's famous statements and Aristotle in his *Physics* tries to refute Zeno's paradoxes of motion.

In my opinion, at least fourteenth-century Oxford natural philosophy can be recognized as a continuation of this tradition. As mentioned above, Richard Kilvington's most popular argument concerning the spiral line, while usually discussed in the commentaries on *Physics*, cannot be taken as anything more than a counter-intuitive geometrical puzzle. And even after John Buridan had resolved it, many medieval philosophers found the discussion on the properties of *linea girativa* puzzling enough to include it in their treatises. Thomas Bradwardine's examination of the properties of infinite sets was without any doubt conducted in order to refute Kilvington's theory. One must remember that the idea that infinite sets are in a sense equal to its infinite subsets, although innovative, counters the Euclidean axiom: *A whole is greater than its part*<sup>4</sup>. Therefore, it is easy to understand why Thomas Bradwardine, himself not only a philosopher and theologian but also a renowned mathematician<sup>5</sup>, took up this challenge.

I think that even the fourteenth-century Oxford philosophers' immense interest in the *science of motion* can be explained in a similar way. Aristotle presented only a vague description of relations between active and passive

<sup>1</sup> Johannes Buridanus, *Utrum linea aliqua gyrativa sit infinita ...*, p. 33.

<sup>2</sup> See e.g. Johannes Mair, *Propositum de infinito Magistri Johannis Maioris*, pp. 20–22 & p. 44.

<sup>3</sup> See G. Colli, *La nascita della filosofia*, Milano 1975.

<sup>4</sup> <Euclides>, *Euclidis Elementorum libri priores XII, Ex Commandini et Gregorii versionibus latinis*, edited by Samuel Episcopus Roffensis, Oxford 1802, Axiomatum (Communis conceptio animi) IX, p. 5: *Totum est sua parte maius*.

<sup>5</sup> E. D. Sylla, Bradwardine Thomas in: *Routledge Encyclopedia of Philosophy*, E. Craig (ed.), vol. 1, London – New York 1998, pp. 863–867.

forces and velocities in local motion. What is more, a simple interpretation of Aristotle's statements can lead to a contradiction, namely, in case of deceleration from a given velocity<sup>1</sup>. Therefore, it was a big challenge to find the proper, i.e. logically consistent, interpretation of Aristotle's theory. As mentioned above, Richard Kilvington developed the concept that was later adopted by Thomas Bradwardine and described as his own achievement<sup>2</sup>. What is important here, in professor Sylla's opinion even Richard Swineshead in his *Liber calculationum* – the treatise representing the peak of the Oxford natural philosophy – used the method invented by Kilvington only in order *to derive surprising or counter-intuitive results and to determine whether or not these must be accepted*<sup>3</sup>.

We must remember that fourteenth-century thinkers still complied with the Aristotelian hierarchy of sciences, where physics was a theoretical science. Therefore, for them to practise natural philosophy meant to carry on mental experiments and the value of specific statements was first of all determined by their logical consistency. And that, I think, is the answer why Oxford Calculators were commonly recognized by their French and Italian followers as logicians rather than natural philosophers<sup>4</sup>.

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<sup>1</sup> E. Jung & R. Podkoński, *Richard Kilvington on Proportions* in: *Mathématiques et théorie du mouvement (XIV–XVI siècles)*, J. Biard, S. Rommevaux (eds), Villeneuve d'Ascq 2008, pp. 90–94.

<sup>2</sup> Thomas Bradwardine, *Tractatus proportionum seu de proportionibus velocitatum in motibus*, edited by H. L. Crosby Jr. in: *Thomas of Bradwardine, His "Tractatus de proportionibus": Its Significance for the Development of Mathematical Physics*, Madison 1955, p. 38.

<sup>3</sup> E. D. Sylla, *The Oxford Calculators*, p. 561.

<sup>4</sup> E. D. Sylla, *The Oxford Calculators*, p. 541.